Modules for Standard ML

I Basic Principles and Terminology

In its simplest form, a module is just a collection of declarations, or in fact a single compound declaration, that is thought of as a meaningful program unit and given an identity, allowing it to be named.

The evaluation of a declaration produces an environment. However, a declaration will, in general, contain free identifiers, which we will call its prerequisites, and its evaluation requires an environment which binds these prerequisites. Thus the meaning of a declaration dec is a function \([\text{dec}]: \text{Env} \to \text{Env}\).

An environment \(E\) supplies a context for the evaluation of a declaration \(\text{dec}\) must satisfy certain constraints. First, it must bind all the prerequisites of \(\text{dec}\) with the appropriate kinds of bindings (i.e., identifiers used as types in \(\text{dec}\) must be bound to types, etc.), and second, the types assigned to values, constructor, and exceptions by \(E\) must permit a correct typing of \(\text{dec}\). (It must be self-consistent as well; e.g., the proper criteria must be assigned to type constructor names, and type expressions must be well-formed.)
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If a declaration (i.e. module) is to be sufficiently self-contained to permit it to be separately compiled (i.e. compiled outside the context of any particular program), then it must be augmented with a full specification of its prerequisites and their required types. This specification takes the form of an environment signature and will be called the inward interface of the declaration. The specification directives of [SML, §4.3] are just such an inward interface. \{Note: Poly spec\}.

The meaning of such an augmented declaration, \(<I\text{-spec}, dec>\) is a function

\[
[<I\text{-spec}, dec>] : \text{Env}_{I\text{-spec}} \rightarrow \text{Env}
\]

where \(\text{Env}_{I\text{-spec}}\) is the "type" of those environments that satisfy the inward specification \(I\text{-spec}\).

The result of this evaluation is an environment that satisfies an outward interface, which will be defined as the environment signature obtained by type-checking the declaration relative to the inward specification. Thus a more precise typing for the declaration would be

\[
[<I\text{-spec}, dec>] : \text{Env}_{I\text{-spec}} \rightarrow \text{Env}_{O\text{-spec}}
\]
where O-spec is the outward interface specification derived by type-checking dec relative to I-spec. The O-spec specifies the identifiers bound by dec.

The result of evaluating a declaration dec in an environment $E_1$, is a new environment $\llbracket \text{dec} \rrbracket (E_1)$ which is typically used to enrich or augment another environment $E_2$ by concatenation, yielding the environment $E_2; \llbracket \text{dec} \rrbracket (E_1)$.

Note that there is no necessary connection between the environment $E_1$ used to evaluate dec and $E_2$, the environment augmented by the resultant environment. [Note: Spec Proposal]

To summarize, a minimal notion of module consists of a declaration together with an environment signature specifying its prerequisites (the inward interface). These together determine the module's outward interface, which is an environment signature specifying the bindings created by the declaration. [Note: Spec directive]

A module is actually an environment generator, i.e. a function which, when applied to an
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appropriate prerequisite environment, produces
an environment satisfying the outward interface.
This resultant environment will be called an
instance of the module. The distinction between
a module as an environment generator and
the environments or instances it generates is
important to keep in mind, since it is
common to suppress this distinction.

(Note: Generator vs Instance.)

We can make a module more self-contained
by specifying exactly where its prerequisite
bindings come from. A natural way to do this
is to indicate that they are bound in some given
module instances. If \( C = \langle I\text{-}spec, \text{dec} \rangle \) and
\( A \) and \( B \) are module instances, then we can
create an instance of \( C \) by

\[
[C](A;B)
\]

assuming that the environment \( A;B \) satisfies the
inward interface \( I\text{-}spec \). This will be the case
if \( O\text{-}spec_A \); \( O\text{-}spec_B \) matches (or includes) \( I\text{-}spec \).

For the purpose of separate compilation, we can
replace the inward interface with a concatenation of
outward interfaces of modules that includes, or "covers" the inward interface (i.e. which specifies all the prerequisite identifiers appropriately).

For compilation, we do not need the module instances A and B, but just their outward interfaces. Any other instances satisfying these outward interfaces would do to create an instance of C. Thus O-spec_A and O-spec_B are a kind of parameter "type" specification for C, which could be reformulated as the following function

\[ C = \lambda A \cdot O\text{-spec}_A, B \cdot O\text{-spec}_B \cdot [\text{dec}](A; B) \]

[Note: Interfaces]
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Dependence and Inheritance

If we define a module instance

\[ C = C(A;B) \]

by evaluating the declaration of \( C \) in the environment \( A;B \), then clearly \( C \) can be said to depend on \( A \) and \( B \), because bindings from \( A \) and \( B \) are used to help define the types, values, and exceptions bound in \( C \) (by giving meanings to the prerequisite identifiers of \( C \)).

There are two forms of dependence: visible and invisible. A binding \( x = v : \sigma \) in \( C \) depends visibly on type bindings in \( A \) or \( B \) whose names occur in \( \sigma \). A hidden dependency occurs when the declaration binding \( x \) refers to prerequisites bound in the context \( A;B \), but the type of \( x \) does not involve type constructors from \( A \) or \( B \). \( C \) is visibly dependent on its context \( A;B \) if any of its bindings are, i.e. if the outward interface \( 0\text{-spec} \) involves types from \( 0\text{-spec}_A ; 0\text{-spec}_B \). Otherwise the dependence is invisible. Note that dependence on values and exceptions can
only be invisible, though value dependencies often give rise to visible type dependencies. Also visible dependencies are statically detectable.

Dependence relationships are most naturally defined at the level of bindings, but the Terminology can be extended to relate modules (or module signatures).

If $C$ visibly depends on a parameter module $A$ (meaning $\text{spec}_C$ mentions types from $\text{spec}_A$), then $C$ (and its instances) are not truly self-contained. The types of certain names bound in $C$ will contain free or uninterpreted type constructor names which should be bound by the instance $A$ on which $C$ depends.

The completeness of $C$ can be restored by causing $C$ to inherit bindings from the parameter $A$. A straightforward way to achieve this is to redefine $C$ as

$$C = \lambda A: \text{spec}_A. A; [\text{dec}](A)$$

with $\text{spec}_C = \text{spec}_A; \text{spec}(\text{dec}_A \text{spec}_A)$. Of course, this causes $\text{spec}_C$ to contain all of binding specs
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in specA, which are not masked by re-bindings
in \text{SPEC}(\text{dec_c}, \text{specA}), and not just the minimal
set of binding signatures needed to close
\text{SPEC}(\text{dec_c}, \text{specA}). However, from a pragmatic
point of view this seems justified because an
environment which binds a type without supplying
any operations relating to that type leaves the
type binding fairly useless (unless operations relating
to that particular type can be supplied by an
outside source).